President- A. S. Draper
with respects of.
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POWER FACTOR INDICATORS.

BY WILLIAM HAND BROWNE, JR.

The introduction of induction motors in factories brings with it a power factor considerably less than unity. Since this requires a larger current for a given power delivered, the total available output of the generating plant is less, and the efficiency of the system is lower than if the load were non-inductive. Further, the wattless component of the current causes increased armature reactions, and consequently seriously affects the regulation of the system.

It therefore may be thought desirable to balance wholly or in part the wattless component of current, due to the induction motors, by the use of synchronous motors or converters, and it then becomes necessary to have some means of knowing when this has been accomplished.

Methods of Determining Balance.—One method of judging the conditions of the system, which is used to some extent, is to place an ammeter in the line carrying the current for both induction and synchronous motors. Then to secure a balance, the excitation of the synchronous motor is changed until the line current is a minimum. This method, while simple, is exceedingly crude, and becomes entirely unreliable when the synchronous motor is fairly well loaded. As the power factor of the system approaches unity under these conditions, a comparatively large change in the excitation of the synchronous motor produces little or no apparent change in the ammeter reading. This is because the characteristic v curve of the synchronous motor, *i. e.*, armature current on a field current base, is quite flat for large loads. The wattless component of current, however, changes very rapidly as will be shown later. The same criticism applies to the use of an indicating wattmeter for securing balance. Here the excitation of the synchronous motor is changed until the volt amperes as found from the ammeter and voltmeter readings are equal to the true watts as indicated by the wattmeter.

To show that both of these methods are unreliable, suppose a small error has been made in reading the ammeter or wattmeter, due to carelessness or to instrumental errors. Fig. 1 illustrates the effect of this error on the wattless component of current, or what is the same thing on sin φ, the inductance factor; K L represents the power factor for different percentage differences between the true and apparent watts, M N is the corresponding inductance factor curve. For an error of 1 per cent. the inductance factor is .14. For an error of 2 per cent. the wattless component of current is nearly 20 per cent. of the total current. That is to say, when the attendant thinks he has secured a balance, the wattless current may still be a very considerable fraction of the whole, and seriously affect the regulation of the generator.

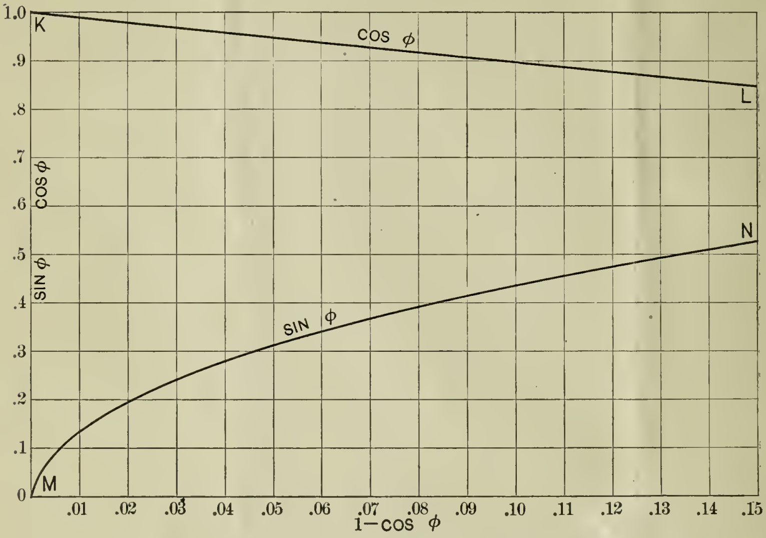

FIG. 1.—Curve showing the rate of change of the sine with the cosine.

Behavior of a Synchronous Motor.—Fig. 2 shows how a synchronous motor will behave under these conditions. Here a constant applied voltage and a constant true watts input have been assumed, with a variable inductance factor. That is to say, $EI \cos \varphi$ is constant, at 5 kilowatts, represented by the horizontal line FA. (Fig. 2).

The base taken here is $EI \sin \varphi$. The curve BC is $\cos \varphi$ for different values of $EI \sin \varphi$. The curve OD shows the corresponding values of $\sin \varphi$; FG is the armature current. These curves show that as we approach unity power factor $\cos \varphi$ changes slowly and the total current changes slowly, but $\sin \varphi$ (and hence $I \sin \varphi$) changes very rapidly.

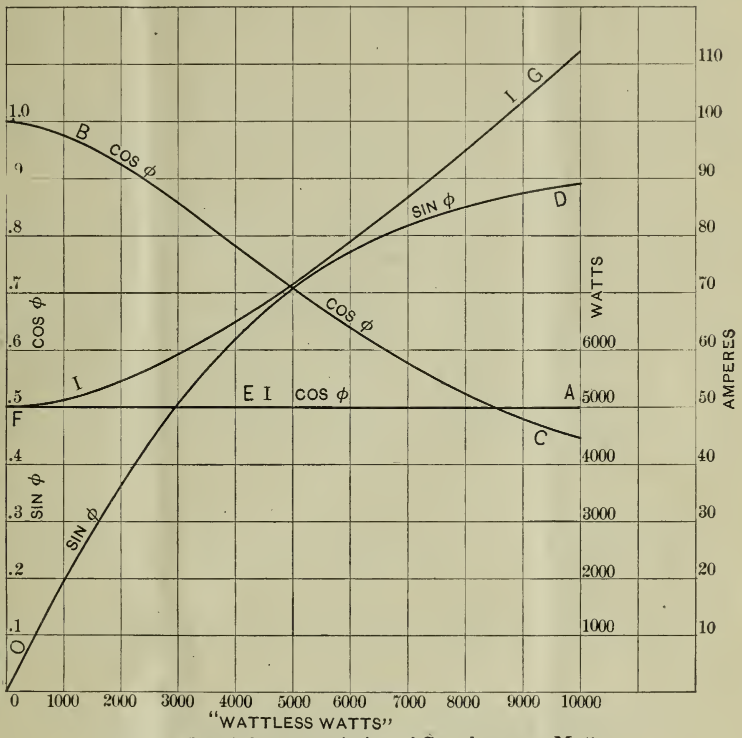

FIG. 2.—Ideal Characteristics of Synchronous Motor.

The need of an instrument which will indicate accurately the condition of the system as regards balance being evident, it becomes worth while examining the methods and instruments available for such determinations, as well as the quantities to be measured. The phase difference of two waves is usually defined as the displacement in degrees between the points where they pass, in the same direction, through zero or their maximum values. In alternating current theory, where the two waves are assumed sinusoids and one is the E.M.F. and the other the current, the cosine of the phase angle is called the power factor. It is equal to the ratio of true watts to volt amperes. In practice while we may not have sinusoids, this ratio is still called the power factor and may be considered as the cosine of the phase angle of the equivalent sinusoids.

As pointed out above, measurements for determining the phase angle are unsatisfactory when involving the cosine, if the cosine approaches unity. We must measure either the phase angle directly or indirectly, or what in most cases would be equally satisfactory, the wattless volt amperes; that quantity Mr. Steinmetz has called the "wattless power."

Unfortunately few of the methods suggested for measuring these quantities are more than laboratory methods, and the instruments used are unsuitable for practical work. The split dynamometer of Blakesley, the three voltmeter method of Fleming, and the three ammeter method of Sumpner are too familiar to need description here, and are hardly applicable under the usual operating conditions. There are, however, two or three instruments which give satisfactory service under proper conditions.

Classification..—Power factor indicators may be divided into four classes:

1. Phase meters, by which the phase angle is determined directly.
2. Power factor meters, measuring a function of the phase angle.
3. Wattless power meters, measuring $E I \sin \varphi$.
4. Wattless current meters, measuring $I \sin \varphi$.

In the first class we may include, in addition to types to be described below, all forms of oscillographs and curve tracers.

In the second class we may include all instruments measuring $\sin \varphi$ and $\tan \varphi$ as well as $\cos \varphi$.

These classes may be further subdivided, according to the means employed to obtain an indication, into four types:

(*a.*) Electromagnetic, using the force of attraction of electromagnets.

(*b.*) Electro-dynamic, utilizing the reaction between coils carrying currents.

(*c.*) Induction, a rotating magnetic field is set up and used to deflect a disk.

(*d*) Electrolytic, depending upon electrolysis to secure a record.

Phase Meters.—The various types of oscillographs and curve tracers are too well known to need description. In general they are laboratory instruments and not applicable to commercial working.

Tuma's Phase Meter[1] (1*b*).—Let the pair of coils A B (Fig. 3) carry the current whose phase angle, referred to the E. M. F. is desired.

Within the space between the two, suspended freely, is the movable system C D, consisting of two independent windings connected together mechanically at right angles. Let a be the angle which D makes with the axis of A B.

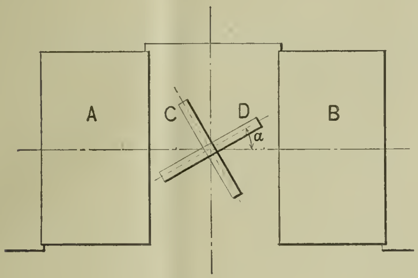

FIG. 3.

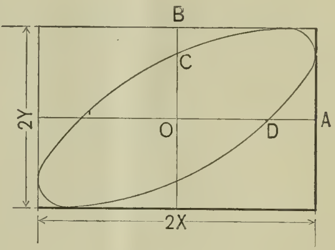

FIG. 4.

When a current $i_1 = I_1 \sin(\omega t - \varphi)$ passes through A B; this sets up an alternating flux in the space between them equal to

$$N = N_0 \sin(\omega t - \varphi)$$

N_0 being the maximum value of the flux and ω 2π times the frequency. Through D pass a current in phase with the E. M. F. This will be

$$i_2 = I_2 \sin \omega t$$

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[1] J. Tuma in *Sitzungsberichte der K. Preussischen Akademie der Wissenschaften*, vol. 106, p. 521, 1897. M. Brieger in *Bulletin de l' Association des Ingenieurs Électriciens*, vol. 11, p. 79, 1899. Aug. J. Bowie, Jr., in *Electrical World and Engineer*, vol. 36, p. 644, 1900. (Mr. Bowie works out the general condition and shows the application to two and three-phase circuits.) H. Armangat in *L'Éclairage Electrique*, vol. 25, p. 339. (Hartmann and Braun instrument.)

And through C pass a current in quadrature with this

$$i_3 = I_3 \sin\left(\omega t - \frac{\pi}{2}\right)$$

The torque set up between A B and D will be

$$T_1 = N i_2 \sin a$$

Between A B and C

$$T_2 = N i_3 \cos a$$

Substituting for N, i_2 and i_3, the values given above, the effective values are

$$T_1 = \frac{N_0 I_2}{2} \sin a \cos \varphi$$

$$T_2 = \frac{N_0 I_2}{2} \cos a \sin \varphi$$

Since the coil is free to turn, the angle a will be such that

$$T_1 = T_2, \text{ then}$$

$$\frac{N_0 I_2}{2} \sin a \cos \varphi = \frac{N_0 I_2}{2} \cos a \sin \varphi$$

$$\tan a :: \tan \varphi$$

The angle a is the phase angle.

Induction Phase Meters (1*c*).—If we pass through two coils, concentric but set at an angle with each other, currents which set up equal fluxes differing, however, in phase, a rotating magnetic field of elliptical form will in general be created. If the two coils are inclined at an angle which is the supplement of the phase angle, the resulting field will have a constant value. An armature suitably placed in such a field will cause a ray of light reflected from an attached mirror to travel in a circle. In applying this method it is necessary to have the fluxes of equal value and to change the angle between the coils until the ellipse becomes a circle when the phase angle is read off directly. The instruments of Angelmeyer[1], Korda[1], Hess[1], Rossi[2] and one of Arno[3], make use of this principle.

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1. (*a*) "Mesure des Differences de Phase." F. Fontaine in *Bulletin de l' Association des Ingenieurs Electriciens*, Nov. 1899.

2. *L'Eclairage Electrique*, vol 15, pp. 133, 322, 355, 1898.

3. *Ibid*, vol. 21, p. 225, 1899.

Electrolytic Phase Indicator (1*d.*)—A very simple device, due to Janet[1], consists of a metallic drum upon which rest two styli of iron. One of these is connected so as to have a difference of potential between it and the drum which is in phase with the current, the other one which is in phase with the E. M. F. On the drum is stretched a sheet of paper which has been soaked in potassium ferrocyanide and ammonium acetate.

When the difference of potential between either stylus and the drum reaches a certain positive value, electrolysis begins and the paper under that stylus turns blue. Now if the drum be turned, each stylus traces a broken blue line on the paper, each blue mark representing part of a positive half wave. The angular distance from the center of one of these short lines and that of the corresponding mark made by the other stylus, expressed in electrical degrees, is the phase angle. This is true only when the two waves have the same form.

Power Factor Meters (2).—Two harmonic motions acting at right angles and having the same frequency and amplitude but a difference in phase of 90°, will produce, if acting at the same time, a uniform circular motion. If the amplitudes are not the same, the result is an ellipse, the major and minor axes of which are the respective paths of the two harmonic motions.

If the phase difference is not 90° the two axes of the resulting ellipse do not coincide in direction with the two harmonic motions.

Puluy's Power Factor Meter (2*a*).—The principle just described has been made use of by Puluy. Two electro-magnets, through the winding of each of which, one of the currents, the phase difference of which is desired, is passed, act upon two armatures causing them to vibrate in planes normal to each other. A ray of light falling upon a mirror attached to one of these and reflected to a second mirror on the other, and thence to a screen, will describe an ellipse, the shape and position of which is determined by the values of the two currents and the phase difference.

Let X be the amplitude of one vibration and Y that of the other. It is evident that a rectangle, the sides of which are respectively $2\ X$ and $2\ Y$, can be circumscribed about this ellipse, Fig. 4. Now when either one of the magnets is acting alone, a

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[1] See reference (*a*) above.

straight line will be described, 2 X or 2 Y in length. The intersection of these lines will be the center of the ellipse.

The co-ordinates of any point on the ellipse, referred to these two lines, will be

$$x = X \sin \omega t \qquad (1)$$

$$y = Y \sin (\omega t - \varphi) \qquad (2)$$

φ being the difference in phase

In equation (1) let $x = 0$, then

$$\omega t = n \pi$$

n being some whole number.

Substituting in (2)

$$\begin{aligned} y_0 &= Y \sin (n \pi - \varphi) \\ &= \pm Y \sin \varphi = \overline{OC} \text{ (Fig. 4)} \qquad (3) \end{aligned}$$

Again let $y = 0$ in (2), then

$$\begin{aligned} \omega t - \varphi &= n \pi \\ x_0 &= X \sin (n \pi + \varphi) \\ &= \pm X \sin \varphi = \overline{OD} \text{ (Fig. 4)} \qquad (4) \end{aligned}$$

From (3) and (4) it follows that

$$\sin \varphi = \frac{O\,C}{O\,B} = \frac{O\,D}{O\,A}$$

Morland's Power Factor Meter.—In an apparatus described by Mr. Morland[1], the two conductors carrying the two currents pass between the poles of a permanent magnet. Each of these causes a small mirror to vibrate, producing the result just described.

Claude's Power Factor Meter.—Claude[2] places the two electro-magnets in direct opposition and causes them to actuate the same armature. Let the maximum flux set up by each coil be N. The ray of light reflected from the mirror will under the influence of the flux N oscillate along a path in length $K N$, K being a constant. If both coils were acting and if the currents were in phase, the length of this path would be 2 $K N$.

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[1] Apparatus for illustrating change of phase, S. I. Moreland, *Electrical Engineer*, Vol. 26, p. 237. Sept. 8, 1898.

[2] See reference (a) above.

Call this d_1. At any instant the flux due to one coil will be

$$N_1 = N \sin \omega t \tag{1}$$

That due to the other will be

$$N_2 = N \sin (\omega t - \varphi) \tag{2}$$

Writing (1)

$$N_1 = N \sin \left[\left(\omega t - \frac{\varphi}{2}\right) + \frac{\varphi}{2}\right]$$

and (2)

$$N_2 = N \sin \left[\left(\omega t - \frac{\varphi}{2}\right) - \frac{\varphi}{2}\right]$$

and taking the sum we have

$$N_1 + N_2 = 2 N \sin \left(\omega t - \frac{\varphi}{2}\right) \cos \frac{\varphi}{2}$$

and the path to the ray is

$$2 N K \cos \frac{\varphi}{2}.$$

Call this d_2.

Then it follows that

$$\frac{d_2}{d_1} = \cos \frac{\varphi}{2}.$$

It is evident from what has been said above that this method will not be accurate when φ approaches zero.

Rayleigh's Power Factor Meter (2*a*).—Lord Rayleigh's apparatus[1] for measuring phase angles is somewhat similar to the above. The movable loop is replaced by a soft iron needle and the two coils are placed on opposite sides of this. The equation then becomes

$$d_1 + d_2 + 2 \sqrt{d_1 d_2} \cos \varphi = d_3$$

The deflections are measured as in a reflecting galvanometer. Mr. Edwin Place[2] gives the results of a great number of experiments made with an instrument of this type. He shows that the angle found in this way is that between the equivalent sine waves. The cosine of this angle, however, is not given by the ratio of true watts to volt amperes, the true power factor.

Tuma[3] describes an instrument similar to the above but with

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[1] *Philosophical Magazine*, May, 1897.

[2] *Electrical World and Engineer*, May 13, 1899, p. 614.

[3] *Sitzungsberichte der K. Preussischen Akademie der Wissenschaften* (Berlin) vol. 106, p. 442, 1897.

the coils at right angles. One of these lies in the magnetic meridian.

He shows that, with specially wound coils, if φ is the phase angle and ψ the angle between the needle and that coil which lies in the magnetic meridian, then

$$\cos \varphi = \tan 2 \psi.$$

Dynamometer Types.—A Siemens electro-dynamometer[1] having two fixed coils may be used for phase angle measurements if the movable system be replaced by a coil closed upon itself and suspended in the plane of the other two.

1. If a current $i_1 = I_1 \sin \omega t$ be passed through one coil, it will set up an E. M. F. in the movable loop

$$e_1 = - M \omega I_1 \cos \omega' t.$$

2. When the current $i_2 = I_2 \sin (\omega t - \varphi)$ passes through the second coil it will set up in the movable coil an E. M. F.

$$e_2 = - M \omega I_2 \cos (\omega t - \varphi)$$

M is the mutual inductance and is assumed the same in each case.

3. If the two currents are flowing at the same time, we have in the movable coil

$$e_1 + e_2 = - M \omega [I_1 \cos \omega t + I_2 \cos (\omega \quad - \varphi)]$$

Denoting the angles of torsion necessary to keep the movaable coil in its zero position in the three cases by d_1, d_2 and d_3, respectively, and by t the constant of the instrument by K, we have

$$K d_1 = M^2 \omega^2 I^2_{1 \text{ eff}}$$

$$K d_2 = M^2 \omega^2 I^2_{2 \text{ eff}}.$$

$$K \underline{d_2} = M^2 \omega^2 (I^2_{1 \text{ eff}} + I^2_{2 \text{ eff.}} - 2 I_1 I_{2 \text{ eff.}} \cos \varphi)$$

Hence

$$d_1 + d_2 - 2 \sqrt{d_1 d_2} \cos \varphi = d_3$$

and the phase angle is deduced from this relation.

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[1] See reference (a) above.

Arno's "Tangent Phase Meter" (2*b*).—A second power meter of Arno's[1] consists of a Siemens electro-dynamometer with an additional pair of coils closed upon themselves and fastened together at right angles. The pair as a whole is suspended within the two other coils and may be held in any position by a torsion spring. In Fig. 5 let A and B be the two coils of the dynamometer and C and D the pair of short-circuited loops.

Passing the two currents through A and B respectively, the instrument becomes a wattmeter and we have

$$I_1\, I_2 \cos \varphi = k_1\, d_1 \tag{1}$$

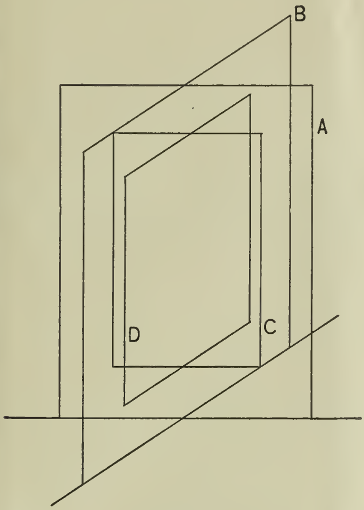

Fig. 5.

where d_1 is the angle of torsion required to keep the movable coil in position. k_1 is the constant for this system.

Now fix B and pass the current I_1 through A, this will set up in C an E. M. F.

$$e_1 = -\, M\, I_1\, \omega \cos \omega\, t \;=\; M\, I_1\, \omega \sin \left(\omega\, t - \frac{\pi}{2}\right)$$

This will cause in C a current

$$i_1 = \frac{M_1}{r_1}\, I_1\, \omega \sin \left(\omega\, t - \frac{\pi}{2}\right)$$

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[1] *L'Eclairage Electrique*, vol. 12, p. 520; vol. 21, p. 226; vol. 25, p. 484; also reference (*a*) above.

The inductance of the loop is assumed negligible; r_1 is the resistance.

This current being in quadrature with that in A will set up no torque.

The current in A will have no inductive effect upon the loop D since they are kept at right angles.

If at the same instant a current I_2 is passed through B, we will have at this instant in B

$$i_2 = I_2 \sin (\omega t - \varphi)$$

$$e_2 = M I_2 \omega \sin (\omega t - \frac{\pi}{2} - \varphi)$$

and a current

$$i^1_2 = \frac{M}{r} I_2 \omega \sin (\omega t - \frac{\pi}{2} - \varphi)$$

This assumes the same mutual inductance for each pair of coils and the same resistance for the two loops C and D.

This current will react with that in A and set up a torque proportional to their product

$$\frac{M I_1 I_2}{r} \omega \sin \omega t \sin (\omega t - \frac{\pi}{2} - \varphi)$$

The effective value of this torque is

$$\frac{M I_1 I_2}{2 r} \omega \cos \left(\varphi + \frac{\pi}{2}\right) = \frac{M I_1 I_2}{2 r} \omega \sin \varphi$$

It is evident that the couple existing between B and C will have the same value but have an opposite sign, since the current in C being in quadrature with that in A will be less than 90° behind B, while that in D will be more than 90° behind A.

The resulting torque of the system is proportional to $I_1 I_2 \sin \varphi$ and may be written

$$I_1 I_2 \sin = k_2 d_2 \qquad (2)$$

where k_2 is a constant and d_2 the angle of torsion.

Dividing (2) by (1) we have

$$\frac{k_1 d_1}{k_2 d_2} = k \frac{d_1}{d_2} = \tan \varphi$$

giving the phase angle in terms of its tangent.

Breitfield's Method (2*b*).—Mr. C. Breitfield[1] has suggested an application of an ordinary wattmeter for measuring the power factor of a three-phase system. The current coil is placed in one line and one end of the pressure coil also connected to this line. The other end is first connected to the second line and then to the third. In the first case we have the deflection

$$d_1 = E\,I \cos(\varphi - 30^\circ)$$

In the second

$$d_2 = E\,I \cos(\varphi + 30^\circ)$$

From these equations we have

$$d_1 - d_2 = E\,I \sin\varphi \sin 30^\circ$$

$$d_1 + d_2 = E\,I \cos\varphi \cos 30^\circ$$

Hence

$$\tan\varphi = \sqrt{3}\left[\frac{d_1 - d_2}{d_1 + d_2}\right]$$

General Electric Company's Instrument (2*b*).—The well-known fact that the ratio of the readings of the two wattmeters used to measure the power of a three-phase system changes with the power factor and is unity when the power factor is unity, has been made use of by the General Electric Company to indicate the power factor. The instrument[2] is simply two wattmeters of the dynamometer type, the movable coils of which are attached to the same spindle. The instrument deflects to the right or left according as the current lags or leads, and stands at zero when the system is balanced and the power factor unity.

Ferraris' Power Factor Meter (2*c*.).—Ferraris[3] combines two harmonic fields, producing a rotating field of elliptical form. Within this field is suspended a short-circuited coil which can be held in any position, by a torsion spring, as is the movable coil of a Siemens dynamometer. The force required and therefore the angle through which the spring must be turned, to hold the coil in any position is proportional to the square of the intensity of the flux in that direction. Points can thus be determined and

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[1] *Electrotechnische Zeitschrift*, vol. 20, p. 120, 1899.

[2] *Electrical World and Engineer*, vol. 37, p. 688, April 27, 1901.

[3] See reference (*a*) above.

the ellipse plotted and the sine of the phase angle deduced as above. (See Puluy's Method, page 481.)

Power Factor Indicators (*3*).—In the methods described above, the object sought was the determination of the phase angle. But few of these are applicable under the usual operating conditions. We might mention as useful forms, those of Tuma and the General Electric Co. In most cases, however, we are more concerned with the magnitude of the wattless component of current than with the power factor. We care less about the decrease in the total output than about the poor regulation caused by inductive loads. Instruments which will indicate the wattless current, or rather the wattless volt amperes are easily made in commercial form.

"*Wattless Wattmeters.*" *Dynamometer Type* (*3b*).—A Siemens dynamometer becomes a wattmeter indicating $E I \cos \varphi$ when the current is passed through one coil and a current proportional to and in phase with the E. M. F. through the other.

If the current in the pressure coil be in quadrature with the E. M. F. the instrument indicates $EI \sin \varphi$, the wattless volt amperes.

This is easily done with a two-phase system. With a single-phase system it is necessary to place a proper condenser or inductive reactance in series with the pressure coil.

"*Wattless Wattmeters.*" *Induction Type* (*3c*). *Dobrowolski's Power Factor Indicator*[1].—In a fully compensated induction meter the shunt flux is in quadrature with the series flux, the result of the two being a rotating or shifting field. If the shunt flux be brought into phase with the series flux an alternating field only will be set up as long as the current and E. M. F. are in phase. If, however, the current lags, the field will rotate in one direction. If it leads, the field will rotate in the opposite direction. The torque developed on a disk placed in this field is, in either case proportional to $E I \sin \varphi$, the wattless volt amperes. This is the principle of Dobrowolski's apparatus.

The movable disk is held in its zero position by a spring when the wattless volt amperes are zero, but deflects one way or the other, when the current is out of phase, an amount proportional to $EI \sin \varphi$.

Any induction wattmeter can be used in this way provided the flux through the pressure coil be brought into phase with the E. M. F.

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[1] U. S. Patent No. 549,449, 1895.

Wattless Current Meters (4).—The Allegemeine Electricitäts Gesellschaft make an instrument[1] which indicates the wattless current. This is really an induction wattmeter having the shunt flux in phase with the E. M. F. and indicates therefore only when there is a phase difference. As an instrument of this type indicates $EI \sin \varphi$, it is presumed that this meter is graduated to read $I \sin \varphi$ at normal voltage, and should properly come under "wattless wattmeters" (*3c.*)

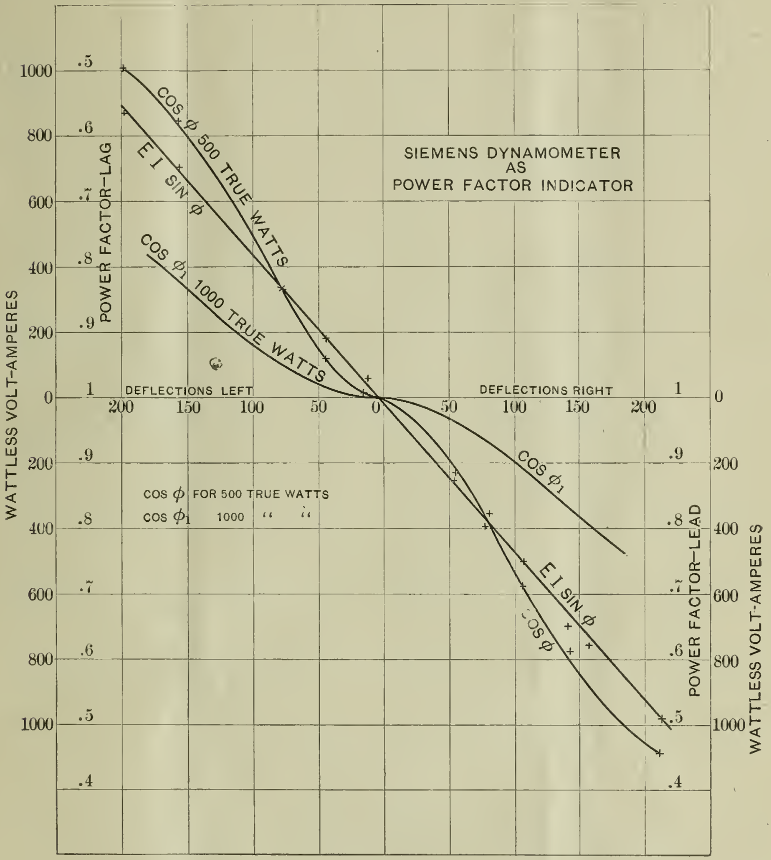

FIG. 6.

BEHAVIOR OF POWER FACTOR INDICATORS.

The Siemens Electro-dynamometer.=The following set of curves (Fig. 6) were taken with a Siemens dynamometer, the

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[1] *L'Eclairage Electrique*, vol. 35, p. 339.

movable coil of which was in series with a capacity of 3.5 microfarads. The values of cos φ, sin φ and EI sin φ plotted have been computed from ammeter, voltmeter and wattmeter readings given in Table I.

EI sin φ is a straight line and is plotted above and below the axis to indicate lagging and leading currents respectively. The base is the deflection in degrees. These curves were taken for a constant power delivered, of 500 true watts. The displacement of current was obtained by means of a phasing transformer. For comparison between EI sin φ and cos φ the axis of abscissæ is taken as unity for the latter, and the decreasing values plotted above and below this.

TABLE. I.

Siemens Dynamometer as Power Factor Indicator.

Amp. I	Volts E	Watts $EI \cos\phi$	Deflection.	Direction.	Phase	Apparent watts EI	Power factors $\cos\phi$	Inductance factors $\sin\phi$	$EI \sin\phi$
10.15	108	500	213	Right	Lead	1096	.456	.889	975
8.4	108	500	158	"	"	907	.552	.834	755
7.58	107.9	500	141	"	"	818	.611	.791	647
6.51	108	500	106	"	"	703	.711	.703	494
5.63	107.8	500	78	"	"	607	.823	.568	294
5.25	107.8	500	53	"	"	516	.884	.467	241
4.78	105.5	500	13	Left	Lag	504	.992	.126	63
5.05	105.7	500	44	"	"	533	.938	.346	185
5.67	106.1	500	76	"	"	602	.831	.555	334
8.17	106	500	155	"	"	866	.577	.817	707
9.61	105.9	500	197	"	"	1007	.497	.867	873

Note.—Pressure coil in series with 3.5 microfarads capacity.

We notice first that these lines do not pass through the origin. The resistance of the pressure coil is not negligible, as the instrument shows a slight deflection at unity power factor. The curves show very clearly that a slight change in the power factor when near unity caused a comparatively large change in the value of EI sin φ.

This fact is very striking when working with the instrument. It may be made to deflect considerably to the right or left without producing any appreciable change in the reading of the Weston wattmeter, the current and voltage remaining constant the while.

Power factors for larger power delivered, lie below the one just considered and hence are still flatter when approaching unity. One for 1,000 true watts has been plotted from calculated values only. It shows that it would be practically impossible to say from ammeter, voltmeter and wattmeter readings alone, when the current was in phase with the E. M. F. With the power factor indicator, however, there would be no difficulty in getting an almost exact adjustment.

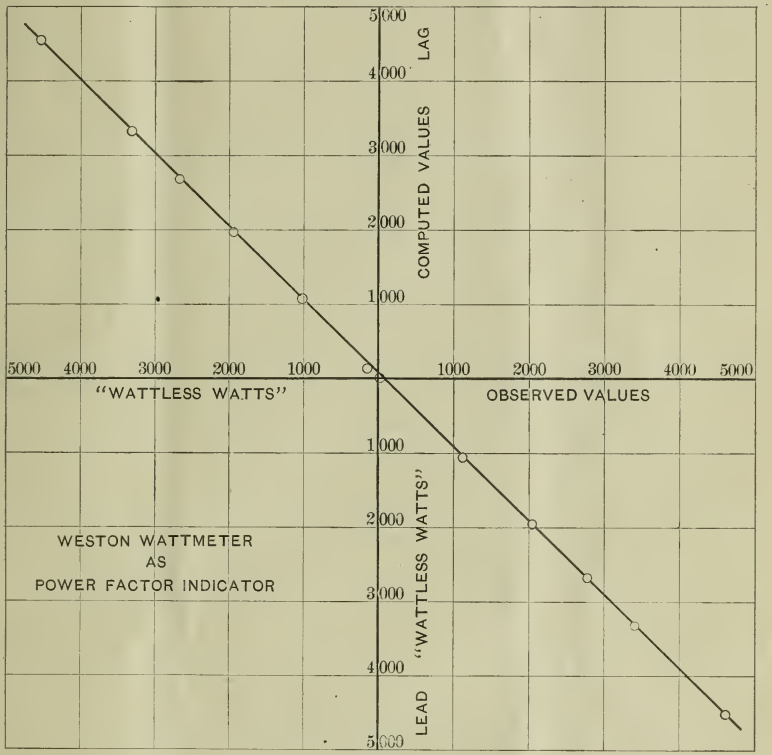

FIG. 7.

Weston Wattmeter as Power Factor Indicator.—In Fig. 7 is given the characteristic of a Weston wattmeter of 150 volts, 50 amperes capacity, used on a two-phase system in the manner described above. The characteristic is a straight line, but does not pass through the origin.

This may be due either to the slight inductance of the pressure coil, or to a slight change in the phase angle between the impressed E. M. F.'s or to both.

The method of calibrating consisted in connecting the pressure coils of the power factor indicator and wattmeter respectively to the two primary windings of a two-phase induction motor, used as a phasing transformer. The current coils of the meters were in series and connected to the movable secondary. As the secondary is shifted, the load is partially shifted from one phase of the primary winding to the other. This doubtless caused a slight change in the phase angle between the E. M. F.'s at the motor terminals. The motor used was rated at two horse-power. The supply was drawn from two 4 K. W. transformers, both partially loaded in addition with lights.

TABLE II.

Weston Wattmeters as Power Factor Indicator.

True Watts.	Voltage.	Amperes.	Wattless volt amperes observed.	Phase relation.	Volt-amperes.	Power factor.	Inductance factor computed.	Wattless volt-amperes computed.
1970	111.5	44.4	4530	Lag	4950	.398	.917	4540
"	"	34.7	3325	"	3870	.509	.860	3330
"	"	29.8	2690	"	3340	.590	.807	2690
"	"	25.0	1950	"	2790	.706	.708	1975
"	"	20.1	1050	"	2240	.879	.477	1070
"	"	17.7	180	"	1975	.997	.071	140
"	"	17.65	0	—	1970	1.0	.0	0
"	"	20.1	1130	Lead	2240	.879	.477	1070
"	"	25.0	2050	"	2790	.706	.708	1975
"	"	29.8	2785	"	3340	.590	.807	2690
"	"	34.7	3420	"	3870	.509	.860	3330
"	"	44.4	4610	"	4950	.398	.917	4540

The agreement between observed and calculated values of $EI \sin \varphi$ is good. Table II gives observed and computed values for this instrument.

Induction Meters.—In Fig. 8 are shown a set of curves obtained from a Shallenberger integrating wattmeter used as a power factor indicator. The instrument is a 500-volt, 10-ampere meter of the older type (1898). The reactive coil was replaced by a non-inductive resistance of 2,000 ohms. In series with this was a capacity of 3 mf., this being found to balance the inductance of the shunt coil at a frequency of 60 cycles. In this case the instrument was used as a recording meter. The curves plotted are

EI sin φ, cos φ and φ, all on a revolutions per minute base, for a delivered power of 500 true watts. Table III gives the instrumental readings and computed values for these curves.

Fig. 9 shows the results from the same meter under somewhat different conditions. Here the recording train was removed to lessen the friction, and a light steel spring attached to the spindle of the revolving disk. A pointer attached to the disk passed over a scale marked in degrees. The curves plotted are, as before, *E I* sin φ, cos φ and φ, for a delivered power of 500 true watts. The base is degrees deflection. It will be noticed that from 20° lag to 20° lead the deflection is almost directly proportional to the phase angle. The flattening of the curve for cos φ is very marked. This instrument was even more sensitive than the dynamometer and besides was dead beat, as the damping

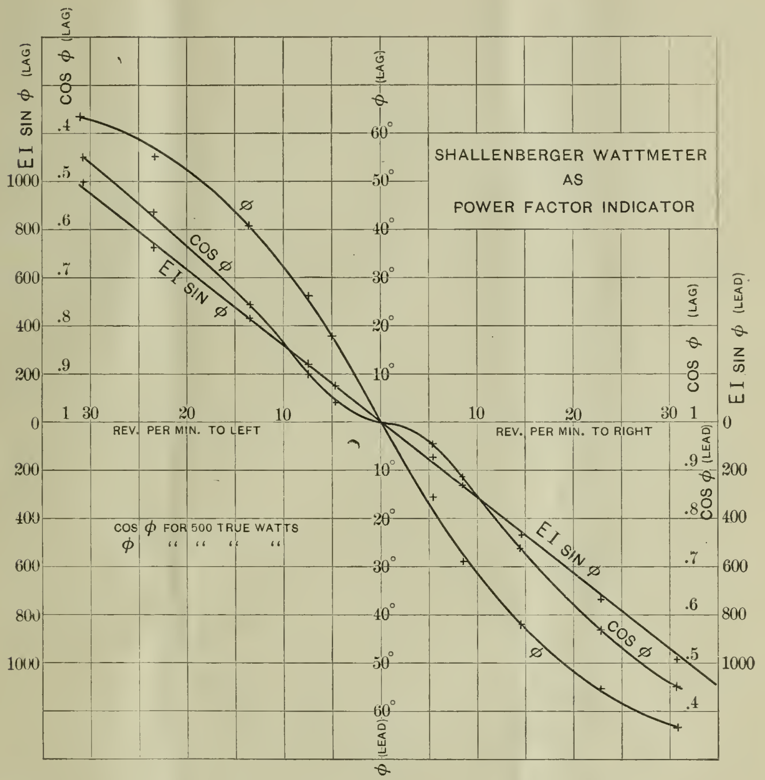

Fig. 8.

magnets were left in. Table IV gives the instrumental readings and computed values for these curves.

Application of Power Factor Indicator.—To illustrate the application of a power factor indicator, the Weston wattmeter mentioned above was used to measure the wattless volt amperes taken by a General Electric 7½-kilowatt, 125-volt, four-pole, synchronous converter for different values of field current. The converter was run on a two-phase system at constant input. The supply was drawn from two Westinghouse 7½-kilowatt transformers, O. D. type, fitted with a Hartford regulator. The output was absorbed by a lamp bank and water rheostat.

TABLE III.

Shallenberger Wattmeter as Power Factor Indicator. 10 amp. 400 volts.

No.	Amp.	Volts.	Watts	r.p.m.	Direction.	Phase.	Apparent watts $E\,I$	Power factor $\cos\phi$	Phase Angle ϕ.	Inductance factor $\sin\phi$	$E\,I\sin\phi$
1	10.01	111.3	505	30.8	Left	Lagging	1114	.449	63° — 10′	.898	996
2	8	111.5	505	23.4	"	"	891	.567	55—27	.824	734
3	6	111.5	505	13.45	"	"	669	.756	40—53	.655	438
4	5.07	111.0	505	7.45	"	"	563	.898	26—06	.440	248
5	4.78	110.3	505	4.68	"	"	527	.954	17—26	.300	158
6	4.5	111.9	505	0	—	—	504	1.00	0	0	0
7	4.58	114.5	505	5.29	Right	Leading	525	.962	15—50	.273	143
8	5.00	114.3	504	8.7	"	"	572	.881	28—14	.473	270
9	6.09	112.2	505	14.6	"	"	684	.738	42—26	.675	462
10	8.0	111.3	505	22.8	"	"	890	.567	55—28	.824	733
11	10.0	111.2	505	30.6	"	"	1112	.454	63—00	.891	992

Note.—Impedance coil cut out and shunt coil in series with three microfarads and 2,000 ohms.

In Fig. 10, armature, current and wattless volt amperes have been plotted as ordinates, the base being field current. The input was 3½ K. W. per phase. The flatness of the current curve alluded to above is quite noticeable here. The characteristic for wattless volt amperes is plotted above and below the axis of abscissæ to indicate lagging and leading currents respectively.

The instruments were quite steady, a condition indicating but little hunting of the armature, although there were no devices on the machine to prevent this phenomenon. The power factor indicator, although exactly like the wattmeter, was even steadier than the latter. There was no difficulty in setting the indicator to read any desired value by adjusting the field current.

It was stated[1] at a recent meeting of the National Electric Light Association, during the discussion of a paper on synchronous converters, that it was practically impossible to operate a synchronous motor or converter at unity power factor. The reasons given were hunting of the armature and dissimilar E. M. F. waves of motor and generator. In the above experiment it was found that the power factor indicator could be set at zero and under these conditions the true watts and the volt amperes were equal. This condition would seem to be that of practically unity power factor.

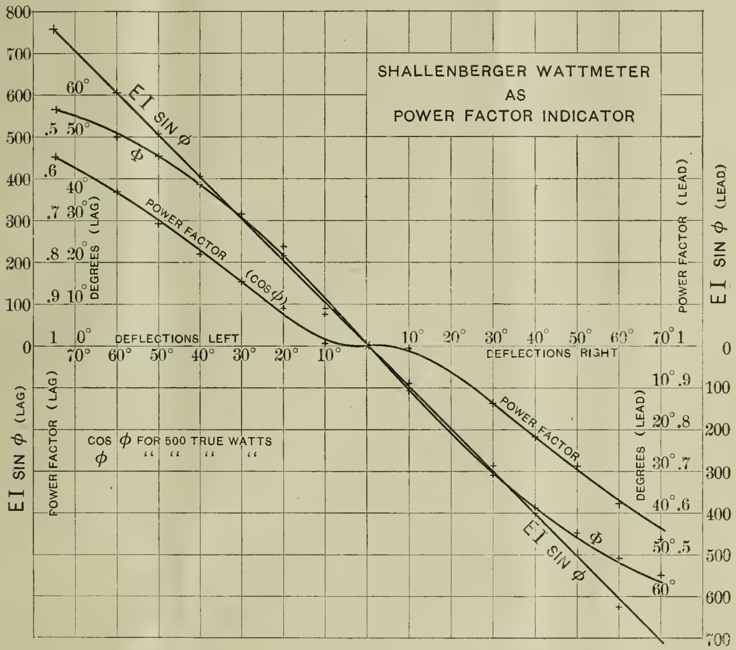

Fig. 9.

A number of power factor curves of synchronous converters have been published recently in which the current passes from lag to lead without the power factor passing through unity. Even if it were impossible to hold the power factor at unity, it must pass through this value as it swings across the line.

In Fig. 11 are plotted, on an excitation base, curves of power factor from wattmeter readings and inductance factor from the

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[1] *Western Electrician*, Feb. 9, 1901.

"wattless wattmeter" readings. In addition there have been plotted two curves obtained by assuming these values to be cosines and sines of phase angles and the corresponding values of sines and cosines taken from tables and plotted on the same base. These deduced values do not agree at all with the observed ones. The discrepancy is greater for leading than lagging currents. This discrepancy, as has been pointed out by Mr. Steinmetz[1], is due to the distortion of the E. M. F. and current waves.

TABLE IV.

Shallenberger Wattmeter as Power Factor Indicator.

No.	Amperes.	Volts.	True watts.	Degrees deflection.	Direction.	Phase	Apparent watts.	cos ϕ	ϕ	sin ϕ	$EI \sin \phi$
1	8.29	110.	500	75°	Left	Lag.	911	.549	56°—42′	.836	761
2	7.27	108.5	500	60	"	"	788	.634	50—39	.773	608
3	6.54	109.	500	50	"	"	712	.702	45—35	.712	507
4	5.85	110.5	500	40	"	"	646	.774	39—17	.633	409
5	5.41	109.2	500	30	"	"	590	.848	32—00	.530	313
6	4.94	109.9	500	20	"	"	543	.921	22—56	.390	212
7	4.57	111.1	500	10	"	"	507	.987	9—04	.158	80
8	4.59	108.4	500	0	—	—	498	1.	0	0	0
9	4.67	109.	500	10	Right	Lead	509	.982	10—53	.189	96
10	5.35	108.	500	30	"	"	578	.865	30—07	.502	290
11	5.87	109.4	500	40	"	"	642	.78	38—44	.626	402
12	6.45	109.7	500	50	"	"	707	.708	45—05	.706	498
13	7.44	107.4	500	60	"	"	799	.626	51—13	.779	623
14	8.16	108.7	500	70	"	"	886	.564	55—40	.826	732

Note.—Impedance coil removed and shunt coil in series with three microfarads and 2,000 ohms non-inductive resistance. Recording gear removed and light torsion spring attached to disk.

Table V gives observed and computed values for these curves.

Conclusions.—It would seem from the above that there is a decided need of an accurate power factor indicator in all large installations, but especially in those in which induction and synchronous motors are used together. This method of operation has been adopted by the Deering Harvester Company, where adjustment of the exciting current of the synchronous machine is

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[1] Symbolic Representation of General Alternating Waves and of Double-Frequency Products. C. P. Steinmetz, TRANSACTIONS, vol. 16, p. 289, 1899.

made by means of a wattmeter, and at Butte, Montana[1], where adjustment is made from ammeter readings. The advisability of adopting this composite system is not in question here. We are considering merely the best method of attaining the end sought.

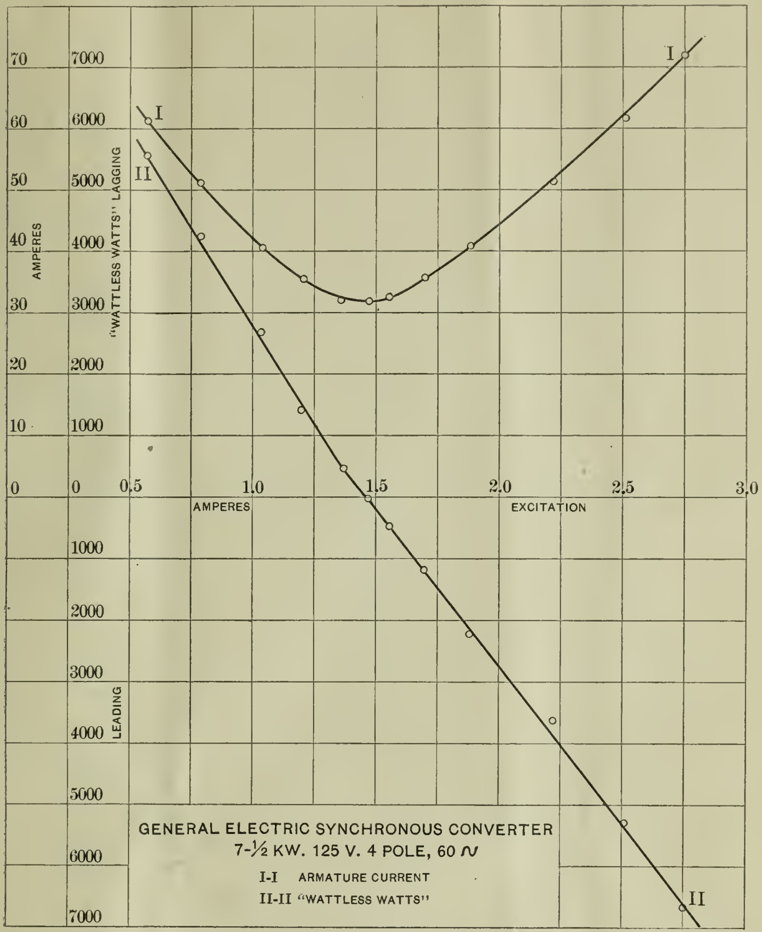

Fig. 10.

For accurate adjusting of the system the indications of the instrument should depend upon the values of φ or sin φ and not upon cos φ.

Instruments of the dynamometer or the induction type seem more suitable for operating conditions. If the meter be of the

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[1] Elec. Lt. & Power at Butte, Mont. J. R. Cravath, Elec. W. & E., vol. 37, p. 149, Jan. 26, 1901.

dynamometer type the resistance of the pressure coil must be negligible if it is to be used on a single-phase system, as the current in this coil must be in quadrature with the E. M. F.

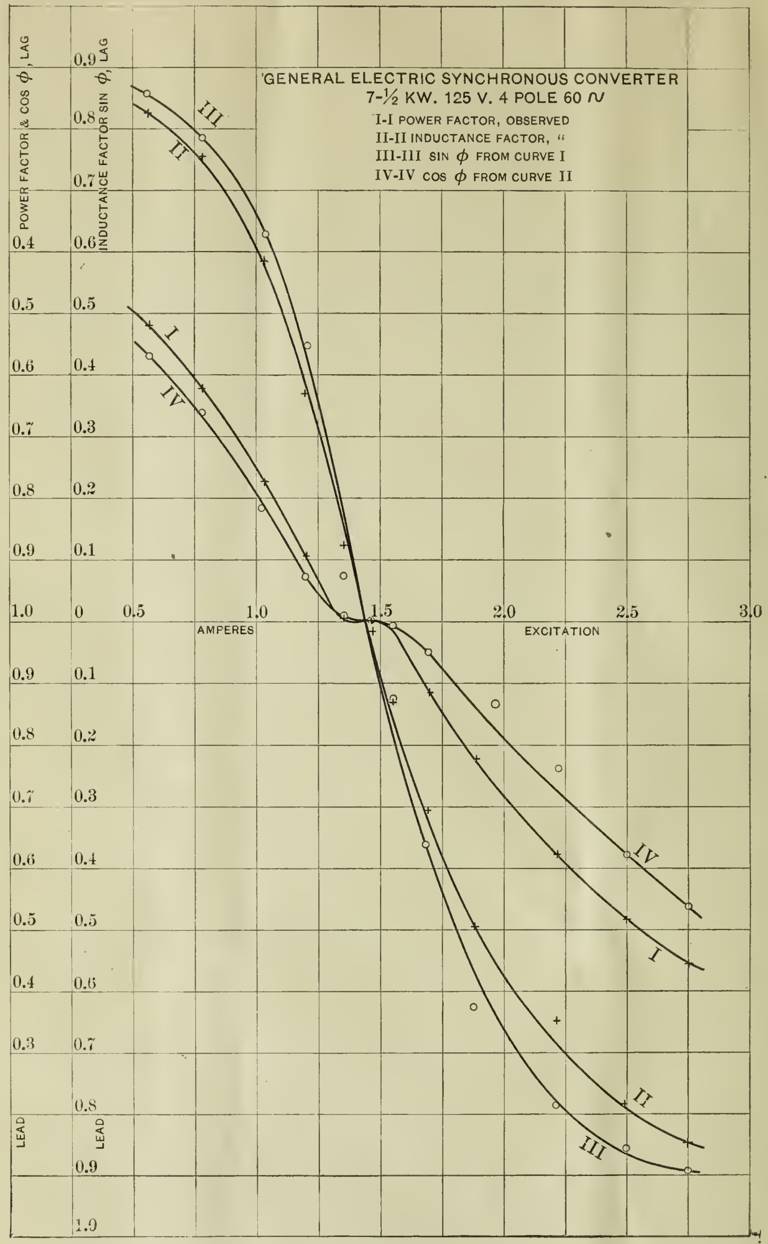

FIG. 11.

If the meter is to be used on a two or a three-phase system, the inductance of the pressure coil must be negligibly small, and meter connected in one of the methods described above. In this case care must be taken that the loads on the different phases be kept equal. In a two-phase system, unbalancing the phases will shift the E. M. F.'s relatively to each other. In a three-phase system, since the meters used are really wattmeters, unbalancing the phases will vitiate the indications of the instrument.

If the meter be of the induction type the inductance of the pressure coil must be negligible, since not only is it undesirable to use condensers to compensate for this, both from their bulkiness and expense, but the presence of any reactance makes the deflection dependent upon the frequency. An attempt was made to improve upon the induction meter described above by removing the three-legged stampings of iron upon which the pressure coil was wound. The inductance was still too large for satisfactory working and as the condensers in the laboratory were not sufficient to compensate for this, it was necessary, to make up for this lack of capacity and to avoid the use of iron, to add an auxiliary inductance wound on a wooden bobbin. The instrument thus modified was found to be so sensitive to slight changes in frequency that it was impossible to use it with any degree of

TABLE V.

General Electric Synchronous Converter. 7½ K.W., 125 V., 4-pole, 60 ~, 2-phase.
Input 3.5 K.W. per phase.

Wattless kilo-volt amperes.	Amperes armature current.	Voltage at brushes.	Amperes field current.	Phase relation.	Kilovolt amperes	Power factor.	Inductance factor.	sin ϕ computed from power factor.	cos ϕ computed from inductance factor.
5.55	61.6	109.9	0.57	Lag	6.76	.518	822	.855	.569
4.23	51.3	110.0	0.78	"	5.64	.621	.750	.784	.661
2.62	40.9	109.9	1.03	"	4.5	.778	.583	.628	.812
1.45	35.7	109.7	1.2	"	3.91	.895	.371	.446	.929
0.45	32.0	109.8	1.36	"	3.51	.997	.128	.077	.992
— 0.05	31.9	109.8	1.47	Lead	3.50	1.0	.014	.0	.999
— 0.45	32.1	110.0	1.55	"	3.53	992	.128	.126	.992
— 1.20	35.7	110.4	1.69	"	3.94	.888	.306	.460	.952
— 2.22	40.9	109.8	1.88	"	4.49	.779	.495	.627	.869
— 3.65	51.3	110.0	2.22	"	5.64	.621	.648	.784	.762
— 5.30	61.6	110.2	2.5	"	6.78	.516	.782	.857	.623
— 6.65	71.9	109.8	2.75	"	7.89	.444	.844	.896	.536

satisfaction in the laboratory. Any change of load on the prime mover would so change the reading of the instrument as to make its indications extremely unreliable.

Dobrowolski shows, in the reference given above, the application of his instrument for automatically adjusting the excitation of the synchronous machine so as to keep the power factor at unity at all times.

The use of an instrument of this kind emphasizes the fact that, when induction motors are used alone, the inductance factor is always a large percentage of the power factor. For instance, when the power factor is .85 the inductance factor, assuming sinusoidal waves, is nearly 62% of this. That is, for every kilowatt used by the motor, 620 "wattless watts" are, so to speak, borrowed. It would seem but fair that those who use induction motors should at least pay rental for the wattless volt amperes required. It is true this does not represent energy consumed, but it does, in a sense, represent energy borrowed and returned and the station must have sufficient capacity in generators to meet all such calls for loans.

The additional charge could be taken care of by over-compensating integrating wattmeters of the induction type, as suggested by Mr. Benischke. They would then read high on lagging reactive loads and low on leading reactive loads. This would put a premium on the use of synchronous motors as they, if not too greatly over-excited, help out in the regulation of the plant. The induction motor has so many points in its favor it can well afford to pay for what it needs—a large wattless component of current.

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